13

Open economy representative agent models

In chapter 10 we explained open economy macroeconomics within a standard macro model of the aggregate demand/supply sort. This model is widely used by macroeconomists for policy and forecasting. Nevertheless it has no explicit micro-foundations and economists have wished to analyse open economy issues in the same way as in the models of the last two chapters. In this chapter we extend some of those models to the open economy. There is a huge degree of complexity possible in such models as a wide array of features special to the open economy can be incorporated (for a full account see Obstfeld and Rogoff, 1996). We limit ourselves rather strictly here and aim to get the reader familiar with some essential elements. In particular we will invariably assume a small open economy facing world markets (one can then easily think of that world economy being effectively a closed economy as in the last two chapters, uninfluenced by its small neighbour).

THE OPEN ECONOMY IN THE CASHLESS OLG MODEL

A natural way to begin thinking about the open economy is in terms of overlapping generations trying to save for their old age. Whereas in chapter 11 this was only possible if the government stepped in with its borrowing, with the open economy it is possible without a government, through the balance of payments. Let there be one sort of perishable good traded throughout the world. We adopt the OLG model of chapter 11 but now allow lending overseas by the young generation, l_t^h, which is then repaid to it when old — as before the old must consume all the resources they have.

The household maximizes the logarithmic utility function

$$U_t^h(c_t^h(t), c_t^h(t+1)) = \ln c_t^h(t) + \ln c_t^h(t+1) \tag{1}$$

subject to $y - \varepsilon - c_t^h(t) = l_t^h$ and $\varepsilon + [1 + r(t)]l_t^h = c_t^h(t+1)$, where $r(t)$ is the world net rate of interest, exogenous to this economy. Its Lagrangean is therefore:

$$J = \ln c_t^h(t) + \ln c_t^h(t+1) + \mu_t^h(t)[y - \varepsilon - c_t^h(t) - l_t^h] \\ + \mu_t^h(t+1)\{\varepsilon + [(1 + r(t)]l_t^h - c_t^h(t+1)\} \tag{2}$$

The first-order conditions yield:

$$c_t^h(t) = \frac{c_t^h(t+1)}{1 + r(t)} \tag{3}$$

The consumer's life-time constraint is:

$$c_t^h(t) + \frac{c_t^h(t+1)}{1 + r(t)} = (y - \varepsilon) + \frac{\varepsilon}{1 + r(t)} \tag{4}$$

so that

$$c_t^h(t) = \frac{y - \varepsilon}{2} + \frac{\varepsilon}{2(1 + r(t))} \tag{5}$$

and

$$l_t^h = \frac{y - \varepsilon}{2} - \frac{\varepsilon/2}{1 + r(t)} \tag{6}$$

We now consider market clearing in the open economy when there is no money, only credit notes denominated in terms of the single good. Our small economy residents can make a loan to foreigners, allowing them to consume home endowments in return for repayment with interest next period; any repaid loans' proceeds will be used on consumption of foreigners' endowments. It is plain that any net loans made will be equal to net home endowments not consumed at home, that is, net exports: we assume in the standard small-economy way that the net loan and net export supplies by our residents make no difference to the world price of loans (the interest rate) or of the single good. So market clearing for our small economy is automatic at world prices via the balance of payments which gives:

$$l_t^h - l_{t-1}^h(1 + r(t-1)) = y - c_t^h(t) - c_{t-1}^h(t) \tag{7}$$

One can imagine these external transactions taking place on the beach where foreigners come to trade; all sales by young residents are settled by credit notes and all credit notes held by old residents are repaid with interest by supplies of goods.

We can now consider the behaviour of our small country's balance of payments on the assumption that the world interest rate is constant at r. With each generation's endowments in youth and old age constant, each young generation will consume and lend the same as the next: hence $l_t^h = l_{t-1}^h = l^h$. Also $c_t^h(t) = c^h$ and $c_t^h(t+1) = c^h(1+r)$ are constant.Since the population of each generation is assumed in (7) to be constant at N, it follows from (7) that the current account of the balance of payments,

$$y - c^h - c^h(1+r) + rl^h = l_t^h - l_{t-1}^h = 0 \tag{8}$$

is in continuous balance.

However this would plainly alter if there was growth, say in population N at the rate n, still assuming constant r. In this case each generation has a constant per capita consumption when young and old, and so also constant borrowing when young — the same as in equation (8). But each generation is $(1+n)$ times the previous one in size.Then the balance of payments equation would become:

$$(1+n)l^h - l^h(1+r) = (1+n)(y - \epsilon - c^h) + \epsilon - c^h(1+r) \tag{9}$$

What this shows is that this country will have a persistent current account balance of payments surplus of, using (6):

$$nl^h = \frac{n}{2}\left\{y - \epsilon\left(1 + \frac{1}{1+r}\right)\right\} \tag{10}$$

This is because the young are always more in number than the old so that their aggregate savings exceed the aggregate dissaving of the old (being only equal when their numbers are equal). It follows by the same argument that if population were declining then the country would be in persistent deficit.

What we see is that the focus of this model is on the dynamics of population and the endowment. The balance of payments is an automatic financing mechanism about which there is no particular concern; its surpluses and deficits will reflect these underlying dynamic factors.

Suppose we return to the constant population/endowment model and examine whether the government could create problems for the balance of payments. The government's budget constraint is:

$$G(t) + L^g(t) = \sum \tau_{t-1}^h(t) + \sum \tau_t^h(t) + [1 + r(t-1)]L^g(t-1) \tag{11}$$

where $\sum \tau_{t-1}^h(t)$ and $\sum \tau_t^h(t)$ are the t-period lump-sum taxes raised respectively on the $t-1$ and t generations.

The household's budget constraint now must deduct the taxes payable from its income in each period:

$$c_t^h(t) = \frac{y - \varepsilon - \tau_t^h(t)}{2} + \frac{\varepsilon - \tau_t^h(t+1)}{2(1+r(t))} \tag{12}$$

and

$$l_t^h = \frac{y - \varepsilon - \tau_t^h(t)}{2} - \frac{\varepsilon - \tau_t^h(t+1)}{2(1+r(t))} \tag{13}$$

Now the balance of payments becomes:

$$L^g(t) + l_t^h - l_{t-1}^h - L^g(t-1) = y - c_t^h(t) - G(t) - c_{t-1}^h(t) + r(t-1)(l_{t-1}^h + L^g(t-1)) \tag{14}$$

with the capital account on the left-hand side of the equal sign and the current account on the right. From this we can see that if the taxes collected on the young and on the old each remain constant, like endowments, population and interest rates, then households will lend a constant amount ($l_{t-1}^h = l_t^h$), and the capital and current accounts will be in balance if the government balances its budget ($L_t^g = L^g(t-1)$). The difference then from the closed economy OLG model is that now the government has no beneficial role in borrowing because the private sector can smooth its consumption without its assistance. Nevertheless of course if the government borrows and so creates a current account deficit, it means it must repay it by reduced spending or extra taxes later; there is no 'balance of payments problem' unless there is a possibility that it will not. But this in turn would be spotted by foreign lenders offering the loans so that what one sees here is a process of smoothing of household and government spending undertaken by foreign lenders in an entirely voluntary, self-enforcing process. It is not difficult to embed temporary or permanent shocks into this model to see the smoothing effect via balance of payments deficits and surpluses.

AN OPEN CASH-IN-ADVANCE MODEL WITH DYNASTIC HOUSEHOLD

The OLG model focuses on longer-term savings decisions and so highlights the role of the balance of payments in smoothing the effects of long-lived shocks to population and GDP. For short-term analysis of the business cycle however it is not much help. For this we naturally turn to one of the models of chapter 12, Lucas' cash-in-advance model, where the household is assumed to be infinitely-lived (that is, to be 'dynastic' in that it cares about the welfare of its descendants). We retain the basic set-up that households, government and foreigners decide in the first half of each period on all financial transactions, including the acquisition of money to buy goods in the home economy and now foreign money to buy foreign goods. We now add a second, foreign, 'country' — the rest of the world. The home and foreign goods are not, as in our OLG model, the same; they compete for the custom of households in the two countries.

Each country has a stock of trees, one per capita. We will allow these trees to be owned by foreigners (capital movements), in fractions, so that in effect 'equity' can be held in someone else's tree — one purchases a fraction of it. This is of no consequence in a closed economy where everyone is identical and so ends up with the same tree; however in the open economy people can buy parts of trees in other countries and as each country has different conditions this may well happen.

The representative household in the home country maximizes in period 0 for example a logarithmic utility function in consumption of home and foreign goods and of leisure:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t + \ln c_t' + \alpha \ln l_t) \tag{15}$$

subject to

$$p_t c_t + \frac{p_t^* c_t'}{e_t} + p_t r_t s_t + \frac{p_t^* r_t^*}{e_t} s_t' = p_{t-1} y_{t-1} + p_t r_t s_{t-1} + \left\{ \frac{p_t^* r_t^*}{e_t} + p_{t-1}^* (1 - l_{t-1}^*)^{\pi} d_{t-1}^* \right\} s_{t-1}' + T_t \tag{16}$$

$$y_t = (1 - l_t)^{\pi} d_t s_t \tag{17}$$

$$m_t \geq p_t c_t \tag{18}$$

$$m_t' \geq p_t^* c_t' \tag{19}$$

where an asterisk denotes 'foreign' and a prime denotes the demand by a home resident for something foreign; c is consumption (hence c' is home consumption of the foreign good); l is leisure; m is demand for home money (m' of foreign money); s is the demand for trees and r is their real price; p is prices; e the exchange rate (foreign currency per unit of home currency); y is output; d is the fruit crop of the tree when the household is working at full stretch ($l = 0$); T is the money transfer from the government.

We use the same sequencing as in chapter 12; the household has to use the proceeds of income in the last period (plus its government transfer and asset disposals) to acquire goods this period; income depends on work, with elasticity π. The consumer makes plans to hold money and assets during the financial sub-period. We assume the leisure plan is then carried out during the production-and-shopping subperiod that follows: shocks to the fruit yield are revealed in this subperiod so that consumption plans may be frustrated by price changes.

Assuming as usual that money's zero nominal return is dominated by the expected nominal return on trees, so that the cash-in-advance constraint is binding, the first-order conditions are:

$$\frac{c'_t}{c_t} = \frac{p_t e_t}{p^*_t} \tag{20}$$

$$E_t \frac{[p_t(1-l_t)^\pi d_t + p_{t+1} r_{t+1}]}{r_t p_t} = E_t \frac{c_{t+1} p_{t+1}}{\beta c_t p_t} \tag{21}$$

$$\frac{\alpha}{l_t} = E_t \frac{\beta \pi p_t d_t (1-l_t)^{\pi-1} s_t}{p_{t+1} c_{t+1}} \tag{22}$$

$$E_t \left\{ \frac{p^*_{t+1} r^*_{t+1} + p^*_t (1-l^*_t)^\pi d^*_t}{p^*_t r^*_t} \right\} \frac{e_t}{e_{t+1}} = E_t \left\{ \frac{p_{t+1} r_{t+1} + p_t (1-l_t)^\pi d_t}{p_t r_t} \right\} \tag{23}$$

the first is the home/foreign goods consumption trade-off; the second that of present versus future consumption; the third that between leisure and future consumption; the last is that between home and foreign trees (uncovered interest parity in nominal terms).

The foreign household has exactly analogous utility and first-order conditions.

The government's budget constraint is:

$$M_t - M_{t-1} = T_t \tag{24}$$

Market clearing gives us:

$$s_t = 1 = s^*_t \tag{25}$$

$$M_t = m_t + m^{*\prime}_t = p_t(c_t + c^{*\prime}_t) = p_t d_t \tag{26}$$

$$M^*_t = m^*_t + m'_t = p^*_t(c^*_t + c'_t) = p^*_t d^*_t \tag{27}$$

$$m_t^{*\prime} + p_t r_t \Delta s_t^{*\prime} - p_{t-1}(1 - l_{t-1})^{\pi} d_{t-1} s_{t-1}^{*\prime} = \frac{m_t'}{e_t} + \frac{r_t^*}{e_t}\Delta s_t' - \frac{p_{t-1}^*(1 - l_{t-1}^*)^{\pi} d_{t-1}^* s_{t-1}'}{e_t} \quad (28)$$

This last is the balance of payments constraint, that foreigners' demand for home money (home exports) and extra home trees minus their earnings from their previous stock of home trees be equal in terms of the same currency to home households' demand for foreign money (home imports) and extra foreign trees minus their earnings from their previous stock of foreign trees. It shows that there is scope for one economy to smooth its consumption for example in the face of a poor crop by selling shares of its trees to foreigners; buying them back later in a good year. This model is in fact rather similar to that of chapter 10, even though the latter had no micro-foundations. It is more complex to solve both because of its non-linearity and because it is a two-country case; to solve it analytically would require either linearising or loglinearising it. (A simpler version without capital flows is solved analytically with some loglinear approximation in Minford, 1995). However our main purpose here is to show its structure.

A SMALL-COUNTRY VERSION

Now let us simplify this to make it possible to derive a tractable analytic solution. First, let us treat leisure as a constant; then let both fruit yield and the money supply be random variables around a constant mean,

$$d_t = \overline{d}(1 + \varepsilon_t) \quad (29)$$

and

$$M_t = \overline{M}(1 + \eta_t) \quad (30)$$

Then rewrite contemporaneous utility as

$$u_t = \ln v_t \text{ where } v_t = \{c_t^{-\rho} + \alpha c_t'^{-\rho}\}^{\frac{-1}{\rho}} \quad (31)$$

$$\text{so that } U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \ln v_t \quad (32)$$

Third, turn this into a small-country model, by assuming that the Rest of the World (the other country) consumes a fixed proportion of the home country's produce, x^*, and does not hold its trees; its prices of goods, p^*, and of trees, r^*, and its tree yield, d^*, are exogenous (and constant).

This implies the new first-order conditions are:

$$\frac{c_t'}{c_t} = (\alpha RXR_t)^{\sigma} \quad (33)$$

$$E_t \frac{(\frac{p_t}{p_{t+1}} d_t + r_{t+1})}{r_t} = \beta^{-1} E_t (\frac{c_{t+1}}{c_t})^{\rho+1} \frac{v_{t+1}}{v_t} \quad (34)$$

$$E_t \frac{[\frac{p_t}{p_{t+1}} d_t + r_{t+1}]}{r_t} = E_t \left(\frac{r^* + d^*}{r^*} \frac{RXR_t}{RXR_{t+1}} \right) \quad (35)$$

where $\sigma = \frac{1}{1+\rho}$ and $RXR_t = \frac{p_t e_t}{p_t^*}$ is the real exchange rate. (34) is uncovered interest parity in real terms, obtained from (23) which is in nominal terms in the manner of chapter 10, equation (9).

The home country's residents now only obtain foreign goods over and above their exports and the return on their foreign trees, by selling some foreign trees, so the balance of payments equation becomes:

$$\frac{m_t'}{e_t} + \frac{r^* p^* (s_t' - s_{t-1}') - d^* p^* s_{t-1}'}{e_t} = m_t^{*\prime} = p_t x^* d_t \quad (36)$$

and so

$$c_t' + r^* \Delta s_t' - d^* s_{t-1}' = RXR_t x^* d_t \quad (37)$$

Market-clearing implies

$$M_t = m_t = p_t(c_t + x^* d_t) = p_t d_t \quad (38)$$

so that

$$c_t = d_t(1 - x^*) \quad (39)$$

and

$$p_t = \frac{M_t}{d_t} \quad (40)$$

as before.

The import function is from (32) and (38):

$$c_t' = (\alpha RXR_t)^{\sigma} c_t = (\alpha RXR_t)^{\sigma} d_t (1 - x^*) \quad (41)$$

It follows that

$$v_t = c_t [1 + (\alpha RXR_t)^{-\sigma\rho}]^{-\frac{1}{\rho}} \quad (42)$$

Equating the right hand sides of (33) and (34), and using (31) and (41), we obtain:

$$\frac{E_t(\frac{1+(\alpha RXR_{t+1})^{-\sigma\rho}}{1+(\alpha RXR_t)^{-\sigma\rho}})^{-\frac{1}{\rho}}(\frac{d_{t+1}}{d_t})^{\rho+1}}{E_t(\frac{RXR_t}{RXR_{t+1}})} = \beta\left(\frac{r^*+d^*}{r^*}\right) \tag{43}$$

Leading this one period forwards but taking expectations at t still, we would find:

$$\frac{E_t(\frac{1+(\alpha RXR_{t+2})^{-\sigma\rho}}{1+(\alpha RXR_{t+1})^{-\sigma\rho}})^{-\frac{1}{\rho}}}{E_t(\frac{RXR_{t+1}}{RXR_{t+2}})} = \beta\left(\frac{r^*+d^*}{r^*}\right) + c_0 \tag{44}$$

where c_0 is (minus) the constant of a Taylor series second order approximation of the left-hand side (in d_{t+1} and d_{t+2}) around $\overline{d}$. From (43) it is plain that RXR must be constant from $t+1$ onwards if we assume for simplicity that $\beta(\frac{r^*+d^*}{r^*}) = 1$, that is, domestic time preference is equal to the world rate of interest) and that c_0 is small enough to neglect. The left-hand side of (43) is unity with a constant RXR; but if RXR were expected to rise (fall) then the left hand side would be greater (less) than unity. To find out this constant value, we can solve backwards from the terminal value (at T, $T \to \infty$) when the transversality condition that $\Delta s_T^{*\prime} = 0$ and $d_T = \overline{d}$ is met; then we can solve from the balance of payments condition (36) that:

$$c_T' = RXR_T x^* \overline{d} + d^* s_{T-1}' \tag{45}$$

Using the import function (40) yields us

$$(\alpha RXR_T)^\sigma (1-x^*)\overline{d} = RXR_T x^* \overline{d} + d^* s_{T-1}' \tag{46}$$

Linearising the left hand side around the presumed equilibrium $\overline{RXR}$ we obtain:

$$\sigma(1-x^*)\overline{d}(\alpha\overline{RXR})^{\sigma-1} RXR_T + f_0 = RXR_T x^* \overline{d} + d^* s_{T-1}' \tag{47}$$

where f_0 is a constant; so that

$$RXR_T = \frac{d^* s_{T-1}' - f_0}{\overline{d}\{\sigma(1-x^*)(\alpha\overline{RXR})^{\sigma-1} - x^*\}} \tag{48}$$

In this we assume that σ is large enough for the denominator to be positive; this is required for stability, as otherwise a rising real exchange rate (relative home to foreign prices) would cause the current account to improve. Since from period $t+1$ the real exchange rate is constant it follows that

$$RXR_{t+i} = \frac{d^* s_{T-1}' - f_0}{\overline{d}\{\sigma(1-x^*)(\alpha\overline{RXR})^{\sigma-1} - x^*\}} \text{ for } i = 1, 2, ..., T \tag{49}$$

Going back to (42), rewrite it as

$$\frac{E_t(1+(\alpha RXR_{t+1})^{-\sigma\rho})^{-\frac{1}{\rho}}}{E_t(\frac{1}{RXR_{t+1}})} = \beta\left(\frac{r^*+d^*}{r^*}\right)d_t^{\rho+1}RXR_t\{1+(\alpha RXR_t)^{-\sigma\rho}\}^{-\frac{1}{\rho}}+c_1$$

where c_1 analogously to c_0 results from the Taylor Series approximation (for the terms in d_{t+1}) around $\overline{d}$. We can see in this equation that with the left-hand side now fixed, a positive shock to GDP lowers the real exchange rate for one period and also by (34) the real return on home trees. Net exports rise (the rise in output raises exports and imports by similar amounts assuming that x^* and α are of similar size; the fall in RXR then assures that imports will fall back from this level), so that foreign trees are acquired. The following period the real exchange rate rises back to a permanent level higher than the pre-shock rate, since foreign stocks of trees are permanently higher.

This example under our assumptions gives the typical boom/bust cycle; because the extra output cannot all be sold abroad more has to be sold domestically which forces people to spend more generally this period (this is effected by the falling real exchange rate driving down the implicit home real interest rate, made up of the world real rate minus the expected appreciation of the real exchange rate).

The role of money in this model is restricted to determining, apart from the current price level, the current real price of trees. Money does not affect the real exchange rate, which is a limitation on this model. We could give it a bigger role by reintroducing a leisure reaction as in the previous model; or we could introduce nominal wage/price rigidity via overlapping wage contracts for example as discussed extensively in earlier chapters for the closed economy. Obstfeld and Rogoff (1996) set out a representative agent model with menu costs of price setting in their last chapter; the characteristics of this model are quite like those of the Dornbusch-style models of chapter 10 above but the complexity required to obtain them is a huge order higher. Increasingly economists are finding that the 'deep structure' models required to model the world in a useful way are far too complex to handle in practice; this then impels them to write down a set of linear approximations, typically of the IS/LM/Phillips-curve form, which are derived from the deep structure model. But of course this was exactly what the original proponents of these models believed they were doing originally — that is, deriving from some theoretical structure some useable macro approximations. We seem to have come full circle! It is good to know one can derive these IS/LM models from micro-foundations but having done so the practical policy maker will want mainly to use models like those of chapter 10 or versions with more elaborate nominal rigidities.

CONCLUSIONS

We have set out here a variety of deep structure open economy models to give an idea of how they are built and what the balance of payments is doing in a deep structure way — viz helping an open economy to smooth consumption and to engage in arbitrage across world markets. Such models are useful exercises for understanding the underlying processes involved in open economy macroeconomics. However in practice except for policy or simulation exercises involving large-scale regime changes it is usual for economists to use the models of the type of chapter 10, which have their origins in the Mundell-Fleming model of the 1960s but have since been 'stretched' to incorporate the modern developments of rational expectations, supply functions, and nominal rigidities in wage-price setting.